

III. CONTINUOUS PROBABILITY

DISTRIBUTION.

There are 3-types of distributions.

1. Normal Distribution.

2- Exponential Distribution.

3. Gamma Distribution.

1. Normal Distribution is Binomial distribution, poission distribution are discrete distribution and but normal distribution is a continuous distribution.

A continuous random variable x is said to be normal distribution with parameters μ and σ^2 if its density function is given by $f(\mu, \sigma, x) = f(x)$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma}\right)^2}$$

where

$$-\infty < x < \infty, -\infty < \mu < \infty, \sigma > 0$$

* The mean of the normal distribution is μ and variance of the normal distribution is σ^2 .

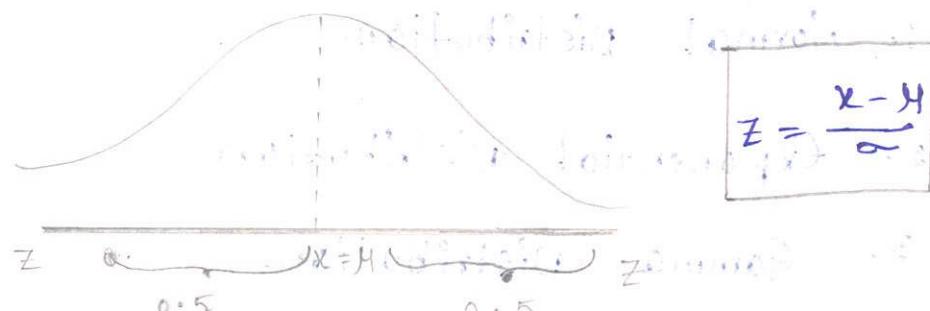
The mean, median and mode of the normal distribution is μ .

$$\therefore \text{mean} = \text{median} = \text{mode} = \mu$$

The maximum probability at $x=\mu$ is $\frac{1}{\sigma\sqrt{2\pi}}$.

The graph of the normal distribution $y = f(x)$ in the xy -plane is known as a normal curve.

The curve is a bell shaped curve.



With this, $Z < 0$ is called $Z \geq 0$ (positive deviation) and $Z > 0$ (negative deviation).

i) For a normal variant with mean 1 and st. deviation 3 find the probability that

$$(i) 3.43 \leq x \leq 6.19 \quad (ii) -1.43 \leq x \leq 6.19.$$

Given that mean $\mu = 1$, st. deviation $\sigma = 3$.

(i) $x = 3.43, \mu = 1, \sigma = 3$

$$z = \frac{x-\mu}{\sigma} = \frac{3.43-1}{3} = 0.81 > 0 \quad (z_1)$$

$$z_1 > 0$$

Now, $x = 6.19, \mu = 1, \sigma = 3$ so the mean is 1.73

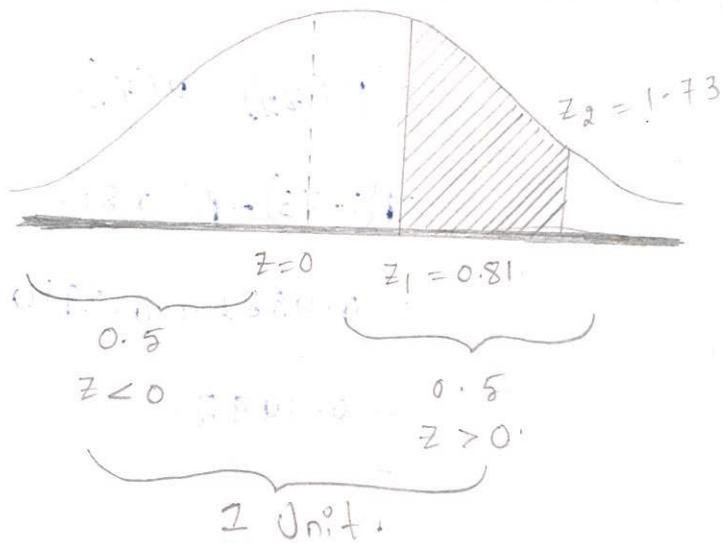
$$z = \frac{x-\mu}{\sigma} = \frac{6.19-1}{3} = 1.73 > 0 \quad (z_2)$$

Now we have to find $P(z_1 \leq z \leq z_2)$

$$P(z_1 \leq z \leq z_2) = P(0.81 \leq z \leq 1.73)$$

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$$P(z_1 \leq z \leq z_2) = P(0.81 \leq z \leq 1.73)$$

$$= P(z_2) - P(z_1)$$

$$= P(1.73) - P(0.81)$$

$$= 0.4582 - 0.2910$$

$$= 0.1672$$

$$(ii) x = -1.43, \mu = 1, \sigma = 3.$$

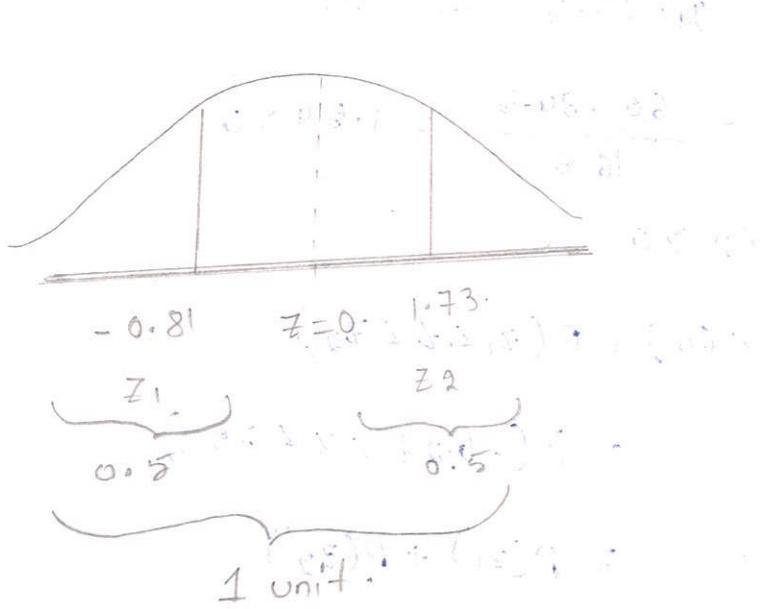
$$z = \frac{x-\mu}{\sigma} = \frac{-1.43-1}{3} = -0.81 < 0$$

$$z_1 = 0$$

$$x = 6.19, \mu = 1, \sigma = 3$$

$$z = \frac{6.19-1}{3} = 1.73 > 0$$

$$z_2 > 0$$



$$\text{Answer: } (-0.81)^2 + (1.73)^2 = \text{unit-3, Pg-3123}$$

$$\begin{aligned}
 P(z_1 \leq z \leq z_2) &= P(-0.81 \leq z \leq 1.73) \\
 &= P(z_2) - P(z_1) \\
 &= P(1.73) - P(-0.81) \\
 &= 0.4582 + 0.2910 \\
 &= 0.7492 \\
 &=
 \end{aligned}$$

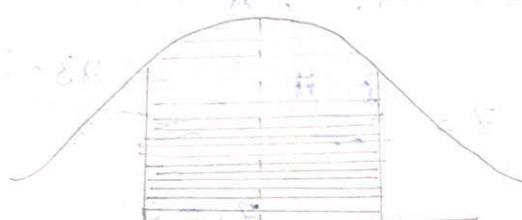
② The mean and standard deviation of the marks obtained by thousand students in an examination are 34.5, 16.5 respectively. If the normality of the distribution, find the approximate no. of students expected to obtain marks b/w 30+60.

$$\text{mean } (\mu) = 34.5$$

$$\sigma = 16.5$$

$$z = \frac{x - \mu}{\sigma}$$

$$x = 30, \mu = 34.5, \sigma = 16.5$$



$$z = \frac{30 - 34.5}{16.5} = -0.27$$

$$z_1 < 0$$

$$x = 60, \mu = 34.5, \sigma = 16.5$$

$$z = \frac{60 - 34.5}{16.5} = 1.54 > 0$$

$$z_2 > 0$$

$$P(30 < x < 60) = P(z_1 < z < z_2)$$

$$= P(-0.27 < z < 1.54)$$

$$= P(z_1) + P(z_2)$$

$$= P(-0.27) + P(1.54) = 0.5446$$

Q. no. of students b/w 30 and 60 is ?

$$0.5466 \times 1000 = 546.6$$

$$546.6 = 547$$

③ Given that mean of height of students in a class is 158 cm with st. deviation of 20cm. Find how many students height b/w 150 cm and 170 cm from if there are 1000 students in the class room.

$$\mu = 158$$

$$\sigma = 20$$

$$x = 150, \mu = 158, \sigma = 20$$

$$z = \frac{x - \mu}{\sigma} = \frac{150 - 158}{20}$$
$$= -0.4$$

$$z_1 < 0$$



$$z_1 = -0.4, \mu = 158, z_2 = 0.6$$

$$x = 170, \mu = 158, \sigma = 20$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{170 - 158}{20}$$

$$= 0.6$$

$$z_2 > 0$$

$$P(150 < x < 170) = P(z_1 < z < z_2)$$

$$= P(-0.4 < z < 0.6)$$

$$= P(z_1) + P(z_2)$$

$$= P(-0.4) + P(0.6)$$

no. of students b/w 150 and 170

$$0.3812 \times 1000$$

$$38.12 = 38$$

=

④ If x is a normal variant with mean 30 and st. deviation 5, Find (i) $26 \leq x \leq 40$

(ii) $x \geq 45$.

$\text{if } \mu=30, \sigma=5 \text{ the required to area under curve}$

(i) $x=26$, $\text{prob. to area under curve}$

$$\text{and we get } z = \frac{x-\mu}{\sigma} \text{ with } \begin{array}{l} z_1 = -0.8 \quad \mu = 30 \quad z_2 = 2 \\ \text{area under curve of standard normal dist.} \\ = \frac{26-30}{5} = -0.8 \end{array}$$

$$z_1 < 0$$

$x=40, \mu=30, \sigma=5$

$$z = \frac{x-\mu}{\sigma}$$

$$= \frac{40-30}{5} = 2$$

$$z_2 > 0$$

$$P(26 \leq x \leq 40) = P(z_1 \leq z \leq z_2)$$

$$= P(-0.8 \leq z \leq 2)$$

$$= P(z_1) + P(z_2)$$

$$= P(-0.8) + P(2)$$

$$= 0.2881 + 0.4772$$

$$= 0.7653$$

$$P(x \geq 45) = P(z \geq z_1)$$

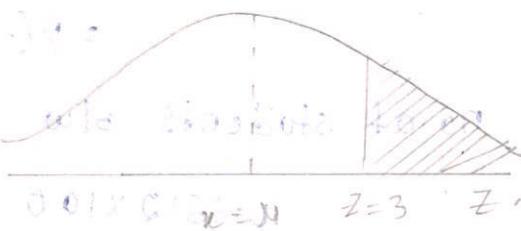
$$= P(z \geq 3)$$

$$= 0.5 - P(z_1) = 0.5 - 0.37$$

$$= 0.5 - P(3)$$

$$= 0.5 - 0.4987$$

$$= 0.0013$$



$$Z = -0.8 \quad Z = 2$$

⑤ If the masses of 300 students are normally distributed with mean 68 kg & standard deviation 3 kg. How many students have masses

(i) Between 65 and 71 kg.

(ii) ≥ 72 kg.

(iii) ≤ 64 kg.

$$\mu = 68, \sigma = 3.$$

$$x = 65$$

$$= \frac{65 - 68}{3} = \frac{-3}{3} = -1.$$

$$(z_1 < 0).$$

$$x = 71$$

$$= \frac{71 - 68}{3} = \frac{3}{3} = 1$$

$$(z_2 < 0)$$

$$P(65 < z < 71) = P(z_1 < z < z_2).$$

$$= P(-1 < z < 1)$$

$$= P(-1) + P(1)$$

$$= 0.3413 + 0.3413 = 0.6826$$

(i) No. of students between 65 and 71 is

$$= 0.6826 \times 300$$

$$= 204.75$$

$$= 205$$

(ii) $x > 72$.

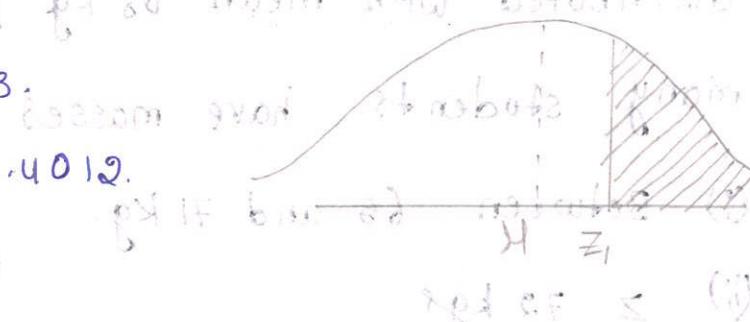
$$x = 72, \mu = 68, \sigma = 3.$$

$$P(z) = \frac{x - \mu}{\sigma} = \frac{72 - 68}{3} = 1.33$$

$$[z_1 > 0]$$

Date _____

Question 3(a) Statistics cos to 202209 part 3C

$$\begin{aligned}
 P(z > z_1) &= P(z > 1.33) \\
 &\text{with pdfs of different & fit 80 norm dist. distribution} \\
 &= 0.5 - 1.33 \\
 &= 0.5 - 0.4012 \\
 &= 0.0918
 \end{aligned}$$


No. of students = 0.0918×300

= $27.54 = 28$

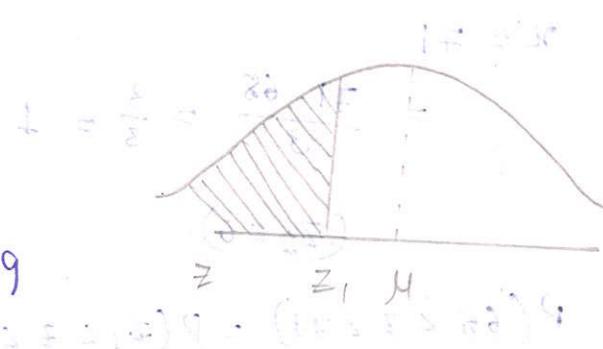
(iii) $x \leq 64$.

$$\begin{aligned}
 z &= \frac{x - \mu}{\sigma} = \frac{64 - 68}{3} = -1.33 \quad (\text{Ans}) \\
 (z_2 < 0)
 \end{aligned}$$

$P(z \leq z_1)$, $P(z < 1.33)$

$= 0.5 - 1.33 = 0.0918$

$= 0.0918 \times 300 = 289$



⑥ In a sample of 1000 cases the mean of a chain certain test is 14 and st. deviation is 2.5 assuming the distribution to the normal.

- (i) Find how many students score b/w 12 and 15
- (ii) How many students score above 18
- (iii) How many students score below 18

$\mu = 14, \sigma = 2.5$

$x = 15, z = \frac{15 - 14}{2.5}$

$x = 12, z = \frac{x - \mu}{\sigma}$

$$\begin{aligned}
 &= \frac{12 - 14}{2.5} = -0.8 \quad (z_1 < 0) \quad (z_2 > 0).
 \end{aligned}$$

$P(12 < z < 15) = P(z_1 < z < z_2)$ [Ans (6)]

$= P(-0.8 < z < 0.4)$

Ques 5 (b) $P(x \geq 18)$ \rightarrow $P(z \geq \frac{18-14}{2.5}) = P(z \geq 1.6)$

$$\begin{aligned} P(z \geq 1.6) &= 1 - P(z \leq 1.6) \\ &= 1 - 0.4435 \\ &= 0.5565 \end{aligned}$$

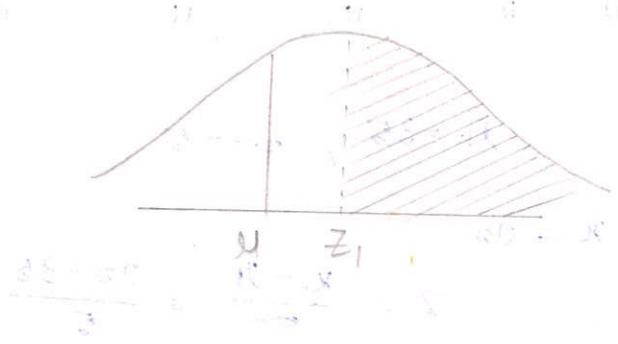
$\Rightarrow 0.5565 \times 1000$ \rightarrow 556.5

(ii) $x < 14$ \rightarrow $P(x < 14) = P(z < \frac{14-14}{2.5}) = P(z < 0)$

$$= 0.5 + 0.4435 = 0.9435$$

$$(ii) x > 18 \rightarrow P(x > 18)$$

$$= \frac{18-14}{2.5} = 1.6$$



$$P(z > z_1) = P(z > 1.6)$$

$$= 0.5 - 0.4435 = 0.0565$$

$$= 0.5 - 0.4452 = 0.0548$$

$$= 0.0548 \times 1000 = 54.8$$

$$= 54.8 = 52 \quad (0.0548 \times 1000) \times 1000 = 54.8 \times 10^4$$

$$(iii) x < 14 \rightarrow P(x < 14)$$

$$z = \frac{x-\mu}{\sigma} = \frac{14-14}{2.5} = 0$$

$$(z_1 > 0) = +1.6$$

$$P(z < z_1) = P(z < 1.6)$$

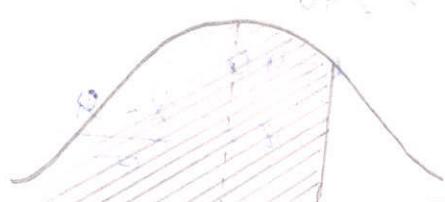
$$= 0.5 + P(1.6)$$

$$= 0.5 + 0.4452 = 0.9452$$

$$= 0.9452 \times 1000$$

$$= 945.2$$

$$= 945$$



$$(0.5 + 0.4452) \times 1000 = 0.9452 \times 1000$$

$$= 945.2$$

⑦ 1000 students have written examination: the mean of test is 35 and st. deviation is 5. Assuming the distribution is normal.

(i) How many students marks lies b/w 25 and 40

(ii) How many students get more than 40

(iii) " " " " " less below 20

(iv) " " " " " more than 50

$$\mu = 35, \sigma = 5$$

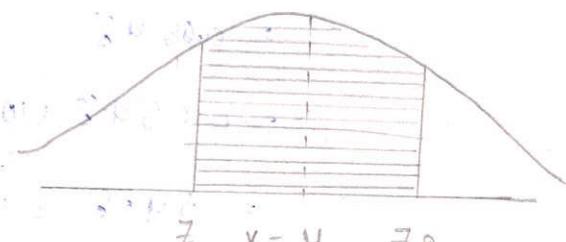
$$x = 25$$

$$z = \frac{x - \mu}{\sigma} = \frac{25 - 35}{5} = -2 (z_1 < 0)$$

$$x = 40$$

$$z = \frac{x - \mu}{\sigma} = \frac{40 - 35}{5} = 1 (z_2 > 0)$$

$$P(z_1 < z < z_2) = P(25 < z < 40)$$



$$= P(-2) + P(0)$$

$$= 0.4772 + 0.3413$$

$$= 0.8185$$

$$\Rightarrow 0.8185 \times 1000 = 818.5$$

$$= 819$$

(ii) $x > 40$

$$x = 40$$

$$z = \frac{x - \mu}{\sigma}$$

$$= \frac{40 - 35}{5} = 1 (z_1 > 0)$$

$$P(z > z_1) = P(z_1 < z)$$

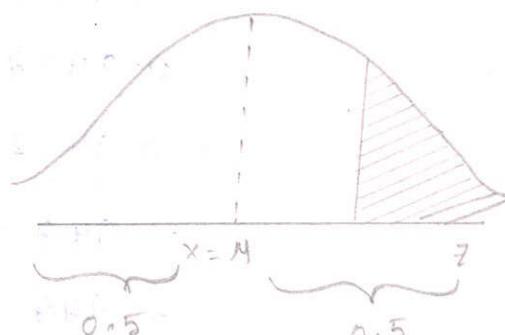
$$= P(0) \neq 0.5 - P(0)$$

$$= 0.5 - 0.3413$$

$$= 0.1587$$

$$0.1587 \times 1000$$

$$= 158.7$$



(iii) $x < 20$

$$z = \frac{x - \mu}{\sigma} = \frac{20 - 35}{5} = -3 \quad (z_1 < 0)$$

$$P(z < z_1) = 0.5 - P(z_1)$$

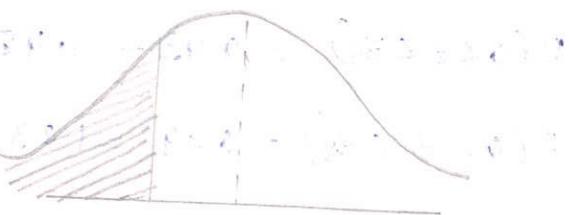
$$= 0.5 - P(3)$$

$$= 0.5 - 0.4987$$

$$= 0.0013$$

$$= 0.0013 \times 1000$$

$$= 1.3 \Rightarrow 1,$$



(iv) $x > 50$

$$z = \frac{x - \mu}{\sigma} = \frac{50 - 35}{5} = 3 \quad (z_2 > 0)$$

$$P(z > z_1) = P(0.5) - P(z_1)$$

$$= 0.5 - P(3)$$

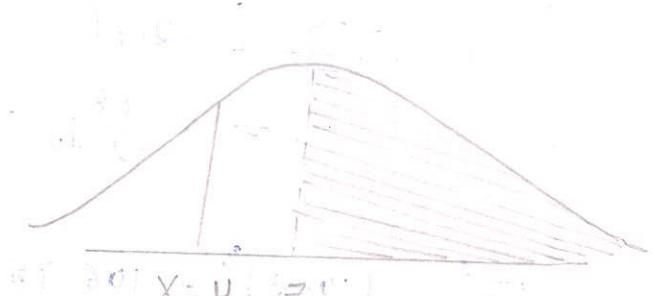
$$= 0.5 - 0.4987$$

$$= 0.0013$$

$$= 0.0013 \times 1000$$

$$= 1.3$$

$$= 1,,$$



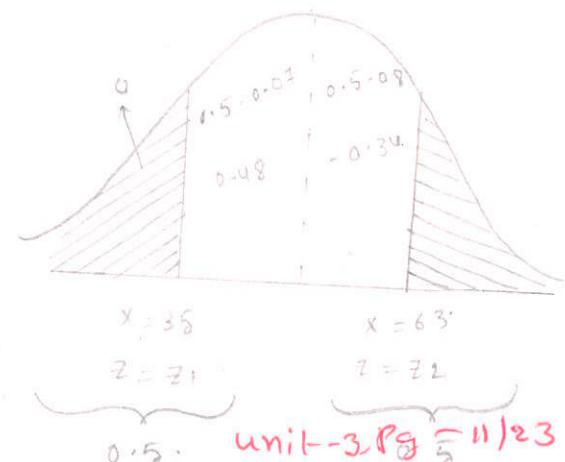
⑧ A normal distribution 7% of items are under 35 and 89% are under 63. Determine the mean and variance of the distribution.

7% under $\rightarrow 35$

89% under $\rightarrow 63$.

$$P(x < 35) = 7\% = \frac{7}{100} = 0.07$$

$$P(x < 83) = 89\% = \frac{89}{100} = 0.89$$



$$P(X \geq 63) = 1 - P(X < 63)$$

$$= 1 - 0.89.$$

$$P(X \geq 63) = 0.11$$

$$P(0 < Z < z_1) = 0.43 = 1.48 = -z_1 \rightarrow ①$$

$$P(0 < Z < z_2) = 0.39 = 1.23 = z_2 \rightarrow ②$$

$$Z = \frac{x-\mu}{\sigma} = \frac{35-\mu}{\sigma} = -z_1 = -1.48 \rightarrow ③$$

$$Z = \frac{x-\mu}{\sigma} = \frac{63-\mu}{\sigma} = z_2 = 1.23 \rightarrow ④$$

③ - ④

$$\left(\frac{35-\mu}{\sigma}\right) - \left(\frac{63-\mu}{\sigma}\right) = -1.48 - 1.23.$$

$$\frac{35-\mu - 63 + \mu}{\sigma} = -2.71.$$

$$\frac{-28}{\sigma} = -2.71.$$

$$\sigma = \frac{-28}{-2.71} = 10.33$$

$$\sigma^2 = (10.33)^2 = 106.75$$

From eqn ③

$$\frac{35-\mu}{\sigma} = -1.48$$

$$\frac{35-\mu}{10.33} = -1.48$$

as we know $\frac{35-\mu}{\sigma} = -1.48 \times 10.33$ from eqn A ③

and we know $\sigma = 10.33$

$$\mu = 50.28$$

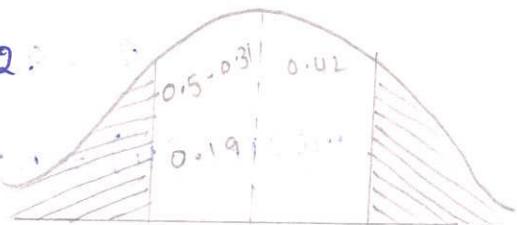
⑨ In a normal distribution 31% of the items are under 45 and 8% of are over 64. Find the mean and variance of the distribution.

$$P(x < 45) = 31\% = \frac{31}{100} = 0.31 \quad \text{--- (1)}$$

$$P(x > 64) = 8\% = \frac{8}{100} = 0.08 \quad \text{--- (2)}$$

$$\begin{aligned} P(x > 64) &= 1 - P(x > 64) \\ &= 1 - 0.08 = 0.92 \end{aligned}$$

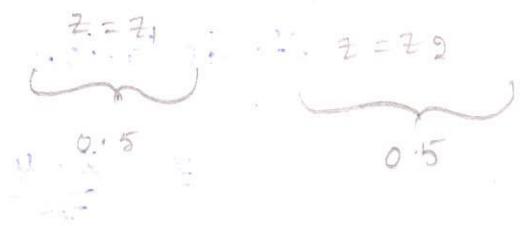
$$P(0 < z < z_1) = 0.19 = 0.5 - 0.31 = 0.19 \quad \text{--- (3)}$$



$$P(0 < z < z_1) = 0.42 = 1 - 0.58 = 0.42 \quad \text{--- (4)}$$

$$\frac{x_1 - \mu}{\sigma} = \frac{45 - \mu}{\sigma} = -0.5 \quad \text{--- (3)}$$

$$\frac{64 - \mu}{\sigma} = 1.41 \quad \text{--- (4)}$$



$$(3) - (4) \quad \left(\frac{45 - \mu}{\sigma} \right) - \left(\frac{64 - \mu}{\sigma} \right) = -0.5 - 1.41$$

$$\frac{45 - \mu - 64 + \mu}{\sigma} = -1.91$$

$$\frac{-19}{\sigma} = -1.91$$

$$\sigma = 9.94$$

$$\sigma^2 = (9.94)^2 = 98.3036$$

$$\frac{45 - \mu}{9.94} = -0.5$$

$$45 - \mu = -0.5 \times 9.94$$

$$45 - \mu = -4.97$$

$$\mu = 49.97$$

(10) Find the probability that out of 100 patients who will live for at least 8 years, 84 and 95 inclusive, will survive a heart operation. Given that chance survival is 0.9

$$n=100, P=0.9.$$

$$q=1-P = 1-0.9 = 0.1$$

$$Z = 1 - 0.9 = 0.1$$

$$\boxed{Z = 0.1}$$

$$\text{mean } (\mu) = np = 100 \times 0.9 = 90$$

$$\text{variance } (\sigma^2) = npq = 100 \times 0.9 \times 0.1 = 9$$

$$\text{st. deviation } (\sigma) = \sqrt{npq} = \sqrt{9} = 3.$$

$$Z = \frac{x-\mu}{\sigma} =$$

$$x=84, \mu=90, \sigma=3.$$

$$Z = \frac{84-90}{3} = -2. (z_1 < 0)$$

$$Z = \frac{x-\mu}{\sigma}$$

$$x=95, \mu=90, \sigma=3.$$

$$Z = \frac{95-90}{3} = 1.66 (z_2 > 0)$$

$$P(84 < Z < 95) = P(z_1 < Z < z_2)$$

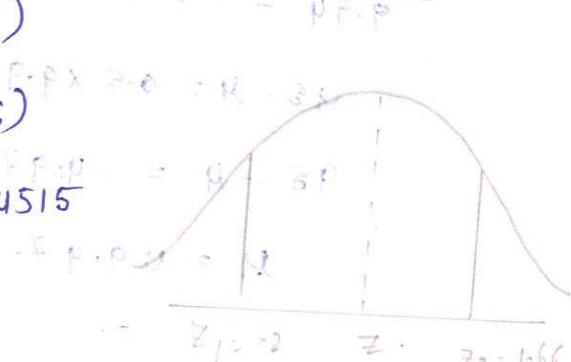
$$= P(-2 < Z < 1.66)$$

$$= P(-2) + P(1.66)$$

$$= 0.4772 + 0.4515$$

$$= 0.9287$$

$$\Rightarrow 0.9287 \times 100 = 92.87 = 93$$



⑪ The marks obtained in statistics in a certain examination found to be normally distributed if 15% of the students ≥ 60 marks and 40% of the students < 30 marks. Find mean and variance and st. deviation.

$$P(X \geq 60) = 15\% = \frac{15}{100} = 0.15$$

$$P(X < 30) = 40\% = \frac{40}{100} = 0.4$$

$$P(0 < Z < z_1) = 0.15 = 0.26 = -z_1 - ①$$

$$P(0 < Z < z_2) = 0.35 = 0.4 = z_2 - ②$$

$$\frac{x-\mu}{\sigma} = \frac{60-\mu}{\sigma} = 1.04 - ③$$

$$\frac{x-\mu}{\sigma} = \frac{30-\mu}{\sigma} = -0.26 - ④$$

$$③ - ④$$

$$\frac{60-\mu}{\sigma} - \left(\frac{30-\mu}{\sigma} \right) = -0.26 - 1.04$$

$$\frac{60-\mu - 30 + \mu}{\sigma} = -1.3$$

$$\frac{30}{\sigma} = -1.3$$

$$\sigma = 23.07$$

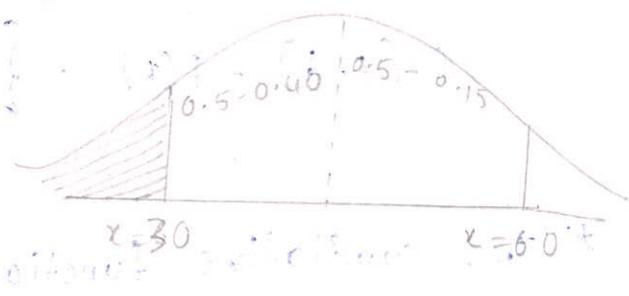
$$\sigma^2 = 532.54$$

$$\text{mean } (\mu) = \frac{60-\mu}{23.07} = 1.04$$

$$60-\mu = 1.04 \times 23.07$$

$$\mu = 60 - 23.9928$$

$$\mu = 36.01$$



Exponential Distribution \sim

A continuous random variable x is said

to be exponential distribution with parameters $\lambda > 0$ if its probability density function is given

by

$$f(x, \lambda) = f(x) = \begin{cases} \lambda e^{-\lambda x}, & x > 0 \\ 0, & \text{Otherwise} \end{cases}$$

- * The parameter λ is called rate parameter it is the inverse of the expected duration.

$$(4) (\lambda = \frac{1}{M}) \text{ (or)} (M = \frac{1}{\lambda}).$$

- * The cumulative distribution function of an exponential distribution is.

$$F(x, \lambda) = F(x) = \begin{cases} 1 - e^{-\lambda x}, & x \geq 0 \\ 0, & \text{Otherwise} \end{cases}$$

- * The cumulative function can be return as the Probability of the lifetime being less than same values of x then

$$P(x \leq x) = 1 - e^{-\lambda x}$$

$$P(x > x) = 1 - P(x \leq x)$$

$$= 1 - (1 - e^{-\lambda x})$$

$$= 1 - 1 + e^{-\lambda x}$$

$$P(x > x) = e^{-\lambda x}$$

Q) Suppose x be an exponential random variable and unit of time is measured in minutes. The probability that the next arrival occurs in

$$(i) \text{ less than } x \text{ minutes } P(X < x) = 1 - e^{-\lambda x}$$

$$(ii) \text{ More than } x \text{ minutes } P(X > x) = e^{-\lambda x}$$

$$(iii) \text{ B/w } x_1 \text{ and } x_2 \text{ minutes } P(x_1 < X < x_2) = e^{-\lambda x_1} - e^{-\lambda x_2}$$

$$\text{Mean } \mu = E(X) = \int_{x=0}^{\infty} x \lambda f(x) dx$$

$$= \int_0^{\infty} x \lambda e^{-\lambda x} dx$$

$$= \text{put } dx = dy \quad \text{---(1)}$$

$$d\lambda = \frac{dy}{dx}$$

$$dy = \lambda dx$$

$$dx = \frac{1}{\lambda} dy$$

$$\text{If } x=0 \text{ then } y=0$$

$$\text{If } x=\infty \text{ then } y=\infty$$

$$= \int_0^{\infty} y e^{-y} \frac{dy}{\lambda}$$

$$= \frac{1}{\lambda} \int_0^{\infty} e^{-y} y dy$$

$$= \frac{1}{\lambda} \left[y e^{-y} \Big|_0^\infty - \int_0^\infty [e^{-y}] dy \right]_0^\infty$$

$$= \frac{1}{\lambda} \left[y e^{-y} - e^{-y} \Big|_0^\infty \right]$$

$$= \frac{1}{\lambda} (0 - (-1))$$

consider $\mu = \frac{1}{\lambda}$ follows the exponential distribution

variance of random variable x will be find below

variance of random variable x will be find below

$$(\rightarrow)^2 = E(x^2) - [E(x)]^2$$

$E(x)$ is the mean of random variable x and μ

$$E(x) = \int_{x=0}^{x=\infty} x^2 f(x) dx$$

$f(x) = \lambda e^{-\lambda x}$ is probability density function of random variable x

$$= \frac{1}{\lambda} \int_0^{\infty} x^2 \lambda^2 e^{-\lambda x} dx$$

$$= \frac{1}{\lambda} \int_0^{\infty} (x-\lambda + \lambda)^2 e^{-\lambda x} dx$$

$$\text{Put } x-\lambda = y \quad \text{then } x = y + \lambda$$

$$dy = \frac{dx}{d(x-\lambda)} dy$$

$$dx = \frac{1}{\lambda} dy$$

If $x=0$ then $y=-\lambda$

$x=\infty$ then $y=\infty$

$$= \frac{1}{\lambda} \int_0^{\infty} y^2 e^{-y} \frac{dy}{\lambda} \quad \text{put } y = -\lambda$$

$$= \frac{1}{\lambda^2} \int_0^{\infty} \frac{y^2}{\lambda^2} e^{-y} dy$$

$$= \frac{1}{\lambda^2} \left[2y^2 \left(\frac{e^{-y}}{-1} \right) - 2y \left(\frac{e^{-y}}{-1} \right) + 2 \left(\frac{e^{-y}}{-1} \right) \right]_0^{\infty}$$

$$= \frac{1}{\lambda^2} \left[-y^2 e^{-y} - 2ye^{-y} - 2e^{-y} \right]_0^{\infty}$$

$$= \frac{1}{\lambda^2} [0 - 0 - 2]$$

$$E(x^2) = \frac{2}{\lambda^2}$$

$$(1-\lambda) \frac{2}{\lambda^2}$$

$$\sigma^2 = E(x^2) - [E(x)]^2$$

(Required value) \neq (Actual value)

$$\text{Actual value} = \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2$$

$$= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} = \frac{1}{\lambda^2}$$

$\boxed{\sigma^2 = \frac{1}{\lambda^2}}$

② The amount of time it takes to make a bank transaction follows an exponential distribution with parameter $\lambda = 0.2$.

- (i) What is the probability that a person weight less than 7 minutes.
- (ii) What is the Probability that a person weight more than 7 minutes
- (iii). What is the Probability that person weight bw 3 and 7 minutes.

$$(i) \lambda = 0.2, x = 3.$$

$$(ii) \lambda = 0.2, x = 7.$$

$$(iii) P(x < x_1 < x_2) = e^{-\lambda x_1} - e^{-\lambda x_2}$$

$$\begin{aligned} \lambda &= 0.2, x_1 = 3 \text{ and } x_2 = 7. \\ &= e^{-(0.2)(3)} - e^{-(0.2)(7)} \\ &= 0.3022. \end{aligned}$$

(or)

$$P(3 < x < 7) = P(x < 7) - P(x < 3)$$

$$= 1 - e^{-0.2 \times 7} - 1 + e^{-0.2 \times 3}$$

$$= 0.3022 //$$

③ A power supply unit for a computer component is assumed to follow an exponential distribution with a mean life of 1200 hrs.

(i) What is the probability that the component will survive $< 1^{st}$ 300 hrs

(ii) Survive more than 1500 hrs? Assume $\lambda = \frac{1}{\mu}$

$$\lambda = \frac{1}{\mu} = \frac{1}{1200} = 0.0008$$

(i) $P(X < 300) = 1 - e^{-\lambda x} = 1 - e^{-0.0008 \times 300}$

$$= 1 - e^{-\frac{1}{1200} \times 300}$$

$$= 1 - e^{-0.002} \approx 0.2211$$

$$(ii) P(X > 1500) = e^{-\lambda x} = e^{-\frac{1}{1200} \times 1500} = e^{-0.0025} \approx 0.2865$$

④ Assume that the length of a phone call in min is an exponential random variable x with parameter $\lambda = \frac{1}{10}$. If same one arrives at phone booth just before you arrive find the probability that you will have to wait.

(i) < 5 min.

(ii) > 5 min.

(iii) b/w 5 and 10 min.

(iv) find mean and variance.

$$(i) \lambda = \frac{1}{10}, x = 5$$

$$= 1 - e^{-\frac{1}{10} \times 5}$$

$$= 0.393$$

(ii) $e^{-\lambda x}$.

$$= e^{-\frac{1}{10} \times 5}$$

$$= 0.606.$$

(iii) $e^{-\lambda x_1} - e^{-\lambda x_2}$.

$$= e^{-\frac{1}{10} \times 5} - e^{-\frac{1}{10} \times 10}$$

= 0.238.

(iv) mean $\frac{1}{\lambda} = 10$.

Variance $\frac{1}{\lambda^2} = 100$.

Q A check out counter at Super marks and completes the purpose process according to an exponential distribution with a survival rate of 6 per hrs.

A customer arrives at the check out counter. find the Probability of the following events.

(i) The service is complete < 5min.

(ii) The customer leaves the check out counter ≥ 10 min after arriving.

(iii) The service is completed in a time b/w 5 and 8 minutes.

$$\lambda = \frac{6}{60} = \frac{1}{10} \text{ hrs per min.}$$

(i) $1 - e^{-(0.1)(5)}$.

$$= 0.3934.$$

(iii) $e^{-0.1 \times 5} - e^{-0.1 \times 8}$.

$$= 0.1572.$$

(ii) $e^{-0.1(10)}$.

$$= 0.3678.$$

Gamma Distribution:

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x} dx.$$

$$\Gamma(1) = 1, \Gamma(\frac{1}{2}) = \sqrt{\pi}.$$

$$\Gamma(n+1) = n \Gamma(n).$$

A continuous random variable x is said to be Γ (Gamma) distribution if its probability density function is.

$$f(x) = \begin{cases} \frac{x^{n-1} e^{-x}}{\Gamma(n)} & x \geq 0, n > 0 \\ 0 & \text{Otherwise.} \end{cases}$$

* Mean and variance of Γ distribution.

$$\text{Mean } (\mu) = E(x) = \int_0^\infty x^r f(x) dx.$$

$$= \int_0^\infty x^r \frac{x^{n-1} e^{-x}}{\Gamma(n)} dx. \quad \text{using formula}$$

$$= \frac{1}{\Gamma(n)} \int_0^\infty x^{r+n-1} e^{-x} dx. \quad (ii)$$

$$= \frac{1}{\Gamma(n+r)} \int_0^\infty x^{n+r-1} e^{-x} dx. \quad (iii)$$

$$= \frac{1}{\Gamma(n+r)} \cdot \sqrt{(n+r)} \cdot \frac{d}{dx} \left[-\frac{1}{\Gamma(n+r)} x^{n+r} \right]_{x=0}^{\infty}$$

$$\mu = E(x^r) = \frac{\Gamma(n+r)}{\Gamma(n)}. \quad (iv)$$

Put $r=1$ in eqn (iv).

$$\mu = E(x) = \frac{\Gamma(n+1)}{\Gamma(n)}$$

$$= \frac{n \Gamma(n)}{\Gamma(n)}$$

$$\mu = E(x) = n.$$

Variance put $r=2$.

$$E(x^2) = \frac{\Gamma(n+2)}{\Gamma(n)}$$

$$= \frac{\Gamma(n+1)+1}{\Gamma(n)}$$

$$= \frac{\Gamma(n+1) + n+1}{\Gamma(n)}$$

$$= \frac{n \Gamma(n) n+1}{\Gamma(n)}$$

$$E(x^2) = n^2 + n.$$

$$\text{variance } (\sigma^2) = E(x^2) - (E(x))^2$$

$$\sigma^2 = n^2 + n - n^2$$

$$\sigma^2 = n.$$

$$\text{mean} = \text{variance} = n.$$

- ① In a class there are 30 students and 6 teachers find the no. of handshakes b/w from excluding the handshakes b/w the teachers.

$$= 36 C_2 - 6 C_2$$

$$= 615$$

=

